

COMMENTS ON “COMBINATORIAL ANABELIAN TOPICS I”

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(1.) In Definition 2.6, (i), “ $\text{Aut}^{|\text{grph}|}(\mathcal{G})$ ” should be defined as the subgroup of $\text{Aut}(\mathcal{G})$ of automorphisms of \mathcal{G} which induce the identity automorphism on the underlying semi-graph of \mathcal{G} [cf. the definition given in Theorem B]. In a similar vein, in Definition 2.6, (iii), “ $\text{Aut}^{|\mathbb{H}|}(\mathcal{G})$ ” should be defined as the subgroup of $\text{Aut}(\mathcal{G})$ of automorphisms of \mathcal{G} which preserve the sub-semi-graph \mathbb{H} of the underlying semi-graph of \mathcal{G} and, moreover, induce the identity automorphism of \mathbb{H} . Since the correct definitions are applied throughout the exposition of the present paper, these errors in the statement of the definitions have *no substantive effect* on the exposition of the present paper, except for the following two instances [which themselves do not have any substantive effect on the exposition of the present paper]:

(i) In the display of Proposition 2.7, (ii), “ $\text{Aut}^{|\text{grph}|}(\mathcal{G})$ ” should be replaced by “ $\text{Aut}^{|\text{VCN}(\mathcal{G})|}(\mathcal{G})$ ”.

(ii) In Proposition 2.7, (iii), the phrase “In particular” should be replaced by the word “Finally”.

(2.) In Definition 2.8, (vii), the phrase “equivalent class” should read “equivalence class”.

(3.) In the portion of the proof of Corollary 5.9 concerning the implication “(iii-3) \implies (iii-1)”, the subscript “ $3g - g + r$ ” should read “ $3g - 3 + r$ ”.

(4.) In the proof of Lemma 1.3, the phrase “scheme-theoretic image of $s_i : B_i \hookrightarrow B \times_k C$ ” should read “scheme-theoretic image of $s_i : B \hookrightarrow B \times_k C$ ”.

(5.) There is a slight [but substantively irrelevant] inaccuracy in the final sentence of the discussion entitled “Curves” in §0. A corrected version of this final sentence may be given as follows:

Then we shall refer to as the *n-th log configuration space* of X^{\log} the log scheme obtained by pulling back the (1-)morphism $\overline{\mathcal{M}}_{g,r+n}^{\log} \rightarrow \overline{\mathcal{M}}_{g,r}^{\log}$ given by forgetting the last n points via the classifying (1-)morphism $T^{\log} \rightarrow \overline{\mathcal{M}}_{g,r}^{\log}$ of $X^{\log} \times_S T$ for some finite étale covering $T \rightarrow S$ [i.e., over which the *divisor of cusps splits*] of

the underlying scheme S of S^{\log} and then descending [in the evident fashion] from $T^{\log} \stackrel{\text{def}}{=} T \times_S S^{\log}$ to S^{\log} .

(6.) In the statement of Theorem C and Corollary 5.10, there are references to [CmbGC], Example 2.5, concerning the definition of a certain *semi-graph of anabelioids of pro- Σ PSC-type* determined by a stable log curve over a base [fs] log scheme whose underlying scheme is the spectrum of a field. At first glance, this may appear to be problematic since the “stable log curve over a base log scheme whose underlying scheme is the spectrum of a field” that appears in [CmbGC], Example 2.5, is subject to the condition that the log structure of this base log scheme be of a certain *restricted type*. In fact, this is not a problem, however, since the construction of the *semi-graph of anabelioids of pro- Σ PSC-type* in [CmbGC], Example 2.5, depends only on the *pointed stable curve* over a field determined by the given log stable curve [i.e., is *independent* of the log structure on the base log scheme].

(7.) There are some flaws in the typesetting of the initial portion of the statement of Corollary 5.10. These flaws did not exist in the original text that was submitted for publication, but apparently appeared at some stage of the typesetting carried out by the publishing company. The [correct] original version of this portion of the statement of Corollary 5.10 is as follows:

Corollary 5.10 (Combinatorial/group-theoretic nature of scheme-theoreticity). *Let (g, r) be a pair of nonnegative integers such that $2g - 2 + r > 0$; Σ a nonempty set of prime numbers; R a complete discrete valuation ring whose residue field k is separably closed of characteristic $\notin \Sigma$; S^{\log} the log scheme obtained by equipping $S \stackrel{\text{def}}{=} \text{Spec } R$ with the log structure determined by the maximal ideal of R ; $x \in (\overline{\mathcal{M}}_{g,r})_S(k)$ a k -valued point of the **moduli stack of curves** $(\overline{\mathcal{M}}_{g,r})_S$ of type (g, r) over S [cf. the discussion entitled “Curves” in §0]; $\widehat{\mathcal{O}}$ the completion of
...*

(8.) At the beginning of Definitions 2.2, 2.4, 2.5, and 2.8, the phrase “at the beginning the present” should read “at the beginning of the present”.

(9.) In the final sentence of the proof of Lemma 1.1, “the fact that the f_i ’s are *isomorphisms* (respectively, ” should read “the fact that the image $\subseteq B \times_k C$ of each section s_i is *of degree 1* over B (respectively, ”.

(10.) In the final display of the proof of Corollary 3.9, the subscript “ b'_i ” that appears above the upper middle horizontal arrow should read “ b'_1 ”.

(11.) In the final sentence of the statement of Lemma 4.3, (ii), the word “satisfies” should be replaced by the text “, which we denote by $Z_{\text{Aut|graph}|(\mathcal{G}_\infty)}(\pi_1^{\text{top}}(\mathbb{G}))$, satisfies”.

(12.) In Remark 5.10.1, (i), the phrase “discussed in [HM],” should read “discussed in [NodNon],”.